

Directed Numbers



Curriculum Ready



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Numbers come in all sizes and forms. They can be positive or negative, whole numbers, fractions or decimals and rational or irrational.

Before you start, investigate these terms and write a brief description about them in the boxes.

Positive and negative numbers:

Whole numbers, fractions and decimals:

Rational and irrational numbers:



Q Calculate the value of: $(-2)^3 + \{-2 + [2 \times (2 + 2)]\}$

Work through the book for a great way to solve this




Types of numbers

Natural numbers are all the positive counting numbers $\{1, 2, 3, \dots\}$.

Integers are all the positive (+) and negative (−) whole numbers (including zero).

Circle all the numbers below that belong to the family known as integers.

(psst!: some of them need to be simplified by calculating their value first)



−1 $\frac{1}{2}$ 13 0.42 −5.0 $\sqrt{7}$ $-\frac{10}{5}$

$\sqrt{4}$ $\frac{4}{5}$ 0 $-\sqrt{2}$ $4\frac{1}{2}$

$\sqrt{5}$ 4 1.19284750219... $-\sqrt{9}$ −16

$\sqrt{36}$ $-\sqrt{8}$ $\sqrt[3]{8}$ 10 9.86

−0.3 $-\frac{1}{3}$ $\sqrt[3]{30}$

Place each of these numbers into their correct group below to classify them.

Irrational numbers can't be written as a fraction

Rational numbers can be written as a fraction

Integers: the positive and negative **whole** rational numbers including 0.

−1

Natural numbers: the counting rational numbers (not including 0).

Put the special types of rational numbers into their correct group



Measuring instruments and integers

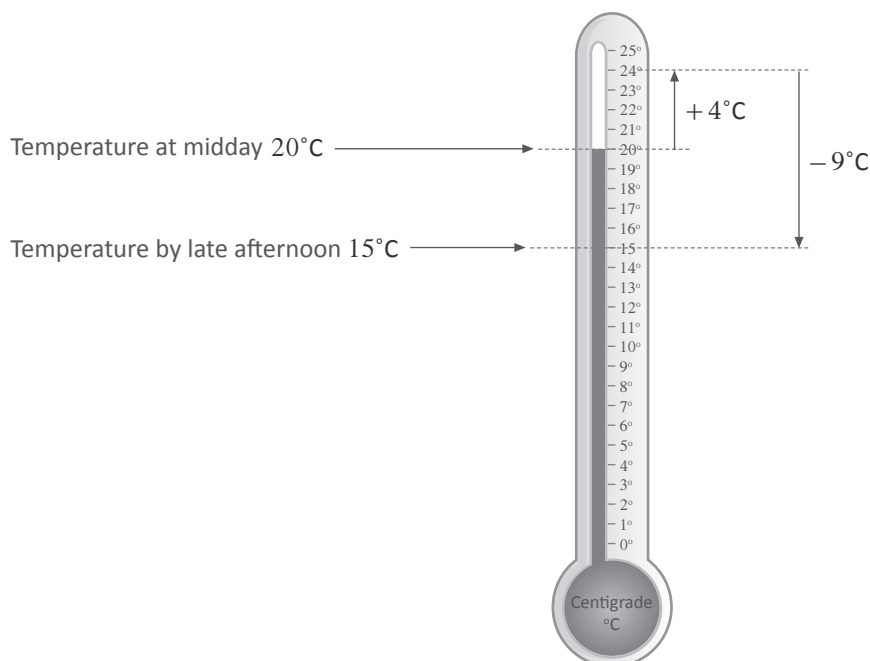
There are many applications for directed numbers.

Thermometers are a great example because the temperature can go below zero.

The temperature at midday was 20 degrees Celsius. During the afternoon, the temperature rose 4 degrees before falling 9 degrees in the late afternoon.

- (i) Shade in the thermometer showing the late afternoon temperature.

Starting at 20°C, temperature changed +4°C then -9°C rose = + and falling = -



- (ii) What directed number gives the overall change in temperature between the two readings?

$$\begin{aligned}\text{Overall change} &= +4^{\circ}\text{C} - 9^{\circ}\text{C} \\ &= -5^{\circ}\text{C}\end{aligned}$$

Digital scales need to use directed numbers to calculate changes in mass.

Use this information to calculate the mass of olives purchased by Maria at her local Deli:

Maria asked for 500g of olives.

After first weighing 575g of olives, the Deli owner took out 98g and then put back 46g.

The directed number sentence for this is: $+575\text{g} - 98\text{g} + 46\text{g}$ Increase = + and decrease = -

$$\therefore \text{Maria purchased } +575\text{g} - 98\text{g} + 46\text{g} = 523\text{g of olives}$$

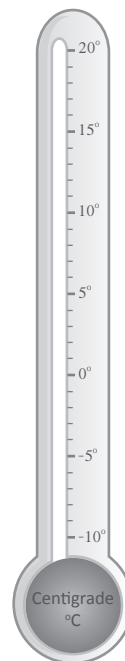
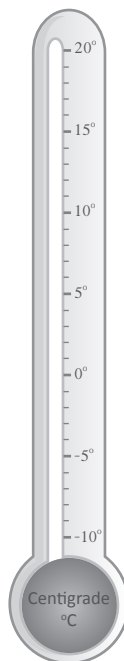
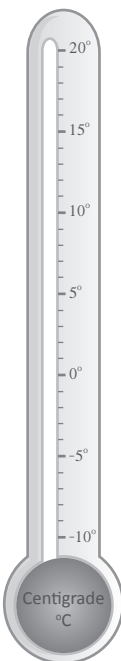
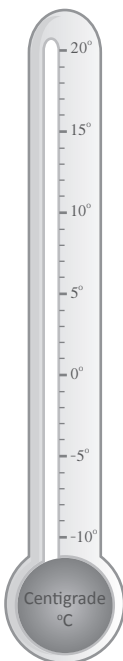
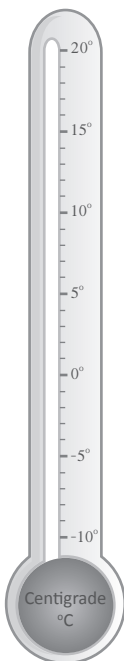
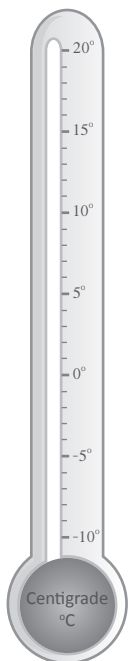
If the first term is positive, we usually don't write in the plus sign.



Measuring instruments and integers

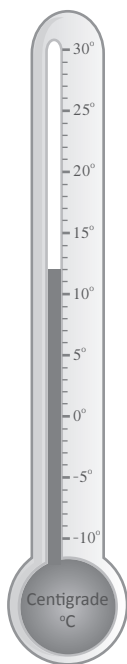
1 Shade in each thermometer below to accurately display these temperatures:

- (i) 11°C (ii) -5°C (iii) 6°C (iv) 0°C (v) 8°C above zero (vi) 2°C below zero



2 This thermometer shows a reading of 12°C at the start.

Write the new readings that would be displayed after each of these changes in temperature:



- (i) Increases by 3°C
- (ii) Becomes 8°C cooler
- (iii) Decreases by 13°C
- (iv) Warms up by 9°C
- (v) Rose by 22°C
- (vi) Dropped 28°C
- (vii) Decrease by 7°C then increase by 12°C
- (viii) Warms by 3°C and then a further 9°C
- (ix) Cools 15°C before warming by 19°C
- (x) Changes to 0°C before dropping 4°C

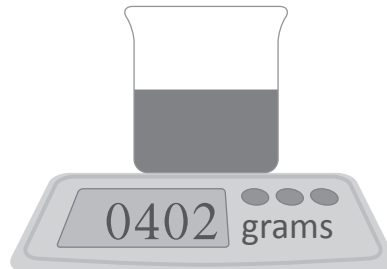
12	$^{\circ}\text{C}$
	$^{\circ}\text{C}$
	$^{\circ}\text{C}$
	$^{\circ}\text{C}$
	$^{\circ}\text{C}$
	$^{\circ}\text{C}$
	$^{\circ}\text{C}$
	$^{\circ}\text{C}$
	$^{\circ}\text{C}$
	$^{\circ}\text{C}$



Measuring instruments and integers



- 3 The mass of a glass container with some water in it is shown below:



Write down the new reading on the scales after any **one** of the following changes occurred.

- (i) 320 g of water poured into the glass

 grams

- (ii) 212 g of water was spilled out of the glass

 grams

- (iii) 88 g of flavour syrup was stirred in and then 250 g was drunk

 grams

- (iv) A similar glass with the same amount of water was placed on the scales next to this one.

 grams

- (v) A cup filled with 216 g of water was emptied into the glass and during the day, 7 g evaporated away because the room was hot.

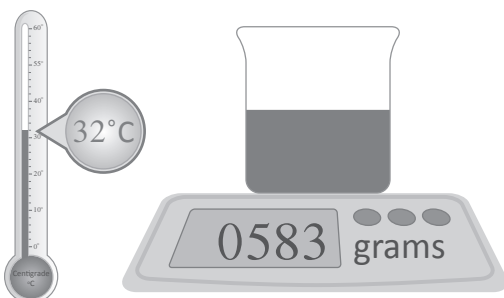
 grams

Try this tricky one!

- 4 A glass container holding **cold** water below was placed on the digital scales after being moved from a 24°C room into a 32°C room.

The glass had collected 3 grams of water (through a process called condensation) on the outside for every 2 degree rise in room temperature.

What was the original mass of the water filled glass if 24 grams of water was spilt during the move?



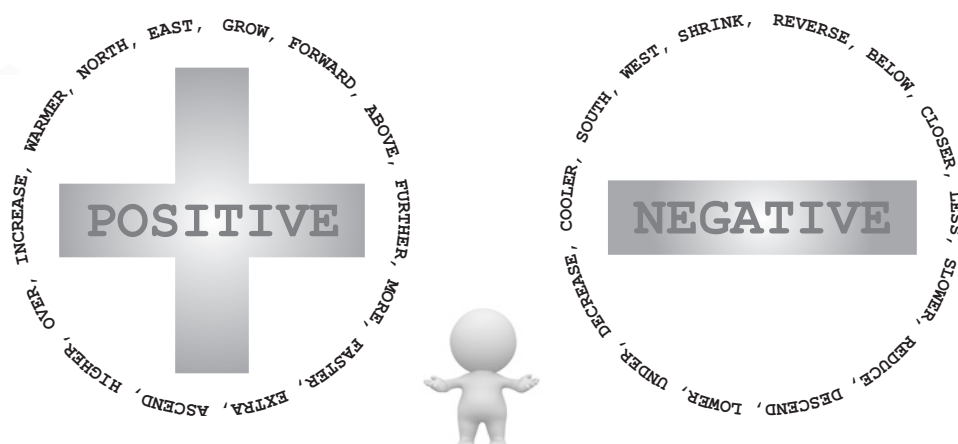
Magnitude and direction of integers

Integers are positive (+) and negative (−) whole numbers, including zeros.
They tell us the size (**magnitude**) and **direction** of values associated with change.



It is important to first decide what will be the positive direction.

- If up (↑) is positive, then the opposite direction down (↓) is negative
- If right (→) is positive, then the opposite direction left (←) is negative
- If hotter is positive (+), then the opposite colder is negative (−)



These are some other words usually associated with positive and negative directed numbers

Only negative numbers must have the sign written in front.

Write down the directed number for each of these statements:

- | | | |
|---------------------------------|--|---|
| (i) 5 degrees below the zero: | $= -5^{\circ}$ | The magnitude of the temperature is 5 degrees |
| (ii) 30 degrees above the zero: | $= +30^{\circ}$ | The magnitude of the temperature is 30 degrees |
| | $= \textcircled{+} 30^{\circ}$ | |
| | ↑
The positive sign (+) can be hidden | |
| (iii) \$12 extra money: | $= +\$12$ | The magnitude of the money is \$12 |
| | $= \$12$ | |

The directed number of an amount left after some changes represents the overall change.

One day Matt started with no money and was given \$40. He spent \$14 of it downloading music.
What is his overall change of money on this day?

The change in money for Matt during this day was: $\$0 + \$40 - \$14 = \26 left

\therefore the overall change in money for Matt on this day is $+\$26$ The final directed value = overall change



Magnitude and direction of integers



- 1 For each of these statements, write down:
(i) the directed number that matches it and, (ii) the magnitude of the directed number.

- a The tree in Jamie's yard grew 3 metres in one year.

(i) Directed number: (ii) Magnitude of the tree's growth:

The same tree was then trimmed which reduced the height of the tree by 4 metres.

(iii) Directed number: (iv) Magnitude of the reduction in height:

- b Aki walked 250 m West from her starting point.

(i) Directed number: (ii) Magnitude of the distance Aki walked:

Aki then turned around and walked another 600 m East

(iii) Directed number: (iv) Magnitude of this distance walked by Aki:

- c Sean's bank account balance earned interest and increased by \$4.

(i) Directed number: (ii) Magnitude of Sean's bank balance increase:

Sean's bank account balance then changed due to being charged \$3 in bank fees.

(iii) Directed number: (iv) Magnitude of the change in Sean's bank balance:

- 2 Think carefully for each of these statements and write down the directed number that matches the **overall change**.

- a Pip was given \$24 and then spent \$15 on a dress the same day.

Directed number for the **overall** change in money that day:

+\$9

- b The temperature initially increased by 6 degrees and then increased by a further 4 degrees.

Directed number for the **overall** change in temperature:

- c Nigel went down 6 rungs on a ladder and then up 2 rungs.

Directed number for the **overall** change in ladder rungs:

- d Cameron's coolness rose by 4 points when he played guitar, and a further 20 points when he did maths.

Directed number for the **overall** change in Cameron's coolness after doing both activities:

- e Shiromee hiked 12 km North, 2 km East and then 19 km South. North is the positive direction.

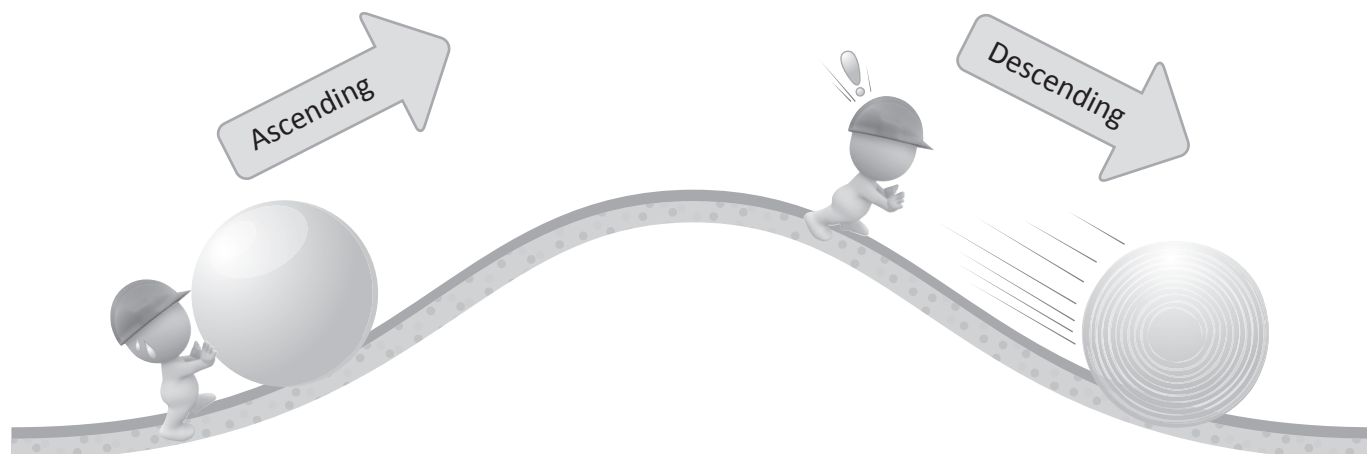
Directed number for the **overall** change in Shiromee's North-South movement:

- f Adele's hair grew from 20 cm to 30 cm long at the back. After a hair cut it was only 17 cm long.

Directed number for the **overall** change in Adele's hair length from the initial 20 cm:

Ascending and descending order

When comparing directed numbers, it often helps to arrange them into numerical order.



Ascending order = lowest to highest

Descending order = highest to lowest

Don't be tricked by the magnitude of a number.

Which of these two values is higher?

2°C

-10°C

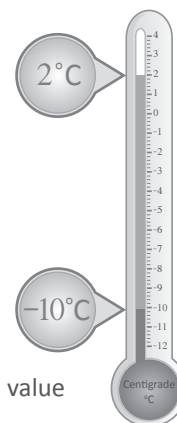
-10°C represents a very cold temperature (**below/less than** 0°C)

2°C is a cold temperature, however it is (**above/greater than** 0°C)

$\therefore 2^{\circ}\text{C}$ is a higher value than -10°C



-10 is a bigger number in magnitude than 2 , however the negative sign makes it lower in value



Let's look at an ascending and descending order example.

For the numbers -4 , 0 , 14.5 , -6 , 5.2 , 3 , -10 , 5.4 :

(i) Arrange them into ascending order

-10 , -6 , -4 , 0 , 3 , 5.2 , 5.4 , 14.5

Lowest negative \longrightarrow Highest positive

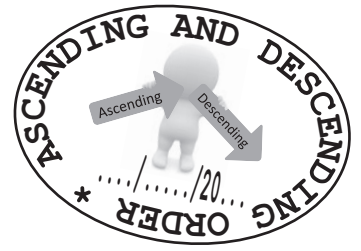
(ii) Arrange them into descending order

14.5 , 5.4 , 5.2 , 3 , 0 , -4 , -6 , -10 Order is opposite to ascending

Highest positive \longrightarrow Lowest negative



Ascending and descending order



1 Circle the word that represents the order of the values in these statements:

- | | | |
|------------------------|------------------|------------|
| a Shortest to Tallest | <u>Ascending</u> | Descending |
| b Longest to shortest | Ascending | Descending |
| c Closest to farthest | Ascending | Descending |
| d Warmest to Coolest | Ascending | Descending |
| e Heaviest to lightest | Ascending | Descending |
| f Thinnest to widest | Ascending | Descending |

2 Arrange the following groups of numbers into **ascending** order (lowest to highest).

- a 14 , 0 , 17 , 3 , , ,
- b 21 , 25 , 20 , 19 , 22 , , , ,
- c -10 , -8 , -12 , -4 , -16 , , , ,
- d 2.2 , 2.4 , -2.6 , 0 , -2 , , , ,
- e -1 , 11 , $\frac{1}{5}$, -1.5 , 1 , $\frac{1}{4}$, , , ,

3 Arrange the following groups of numbers into **descending** order (highest to lowest).

- a 2 , 16 , 5 , 8 , , ,
- b 40 , 31 , 32 , 38 , 29 , , , ,
- c -19 , -11 , -16 , -18 , -13 , , , ,
- d 1.6 , 1.9 , -1.3 , -1.0 , 1.2 , , , ,
- e 0 , $-\frac{1}{4}$, $-2\frac{1}{2}$, -2.1 , 1.2 , -30 , , , ,



Ascending and descending order

Combo Time!

4 Elevator Riding

A department store elevator operator started work on the 3rd floor of the 15 storey building. For the first 15 minutes of work, the operator travelled to the following floors in the order written:



- ☐ 12th floor
- ☐ 8th floor
- ☐ Ground floor
- ☐ 2nd floor
- ☐ Ground floor
- ☐ 15th floor
- ☐ 4th floor
- ☐ 10th floor
- ☐ 5th floor

- a If up is positive and down is negative, write numbers that represent the movement of the elevator operator during the first 15 minutes of work.

.....,,,,,,,

- b Arrange the directed movements into **descending** order.

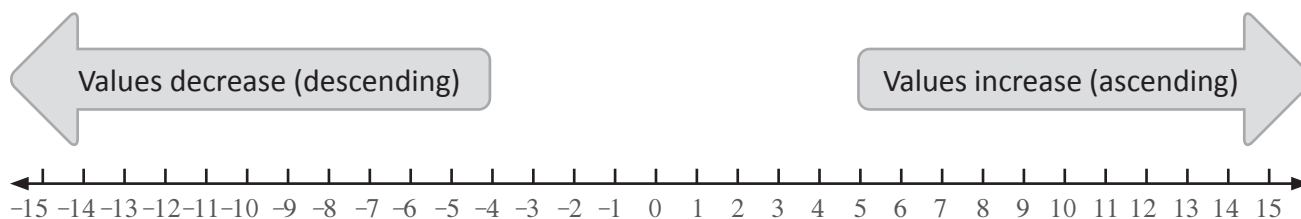
.....,,,,,,,

- c During the first 15 minutes, did the operator mostly ascend or descend in the elevator? Explain your answer.



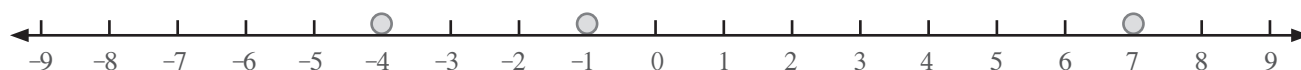
The number line

Directed numbers can be plotted on a number line to instantly see their order of value.



Lower numbers are further left and higher numbers are further right.

Display the numbers -4 , -1 , 7 on a number line.



Use the plotted points to compare the values of:

(i) -4 , -1 : -4 is further left than -1 , $\therefore -4 < -1$

(ii) -1 , 7 : -1 is further left than 7 , $\therefore -1 < 7$



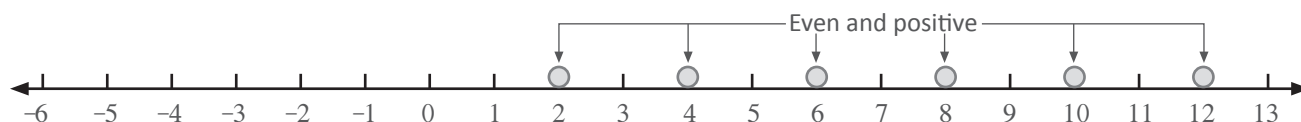
Remember:

$<$ means 'less than' and

$>$ means 'greater than'

This example requests numbers to be plotted using a given rule.

Display all the **even positive integers** between -5 and 13 on a number line.



Write these numbers in **descending** order.

Read the numbers from right to left (\leftarrow)

\therefore The numbers in descending order are: $12, 10, 8, 6, 4, 2$

Half values are plotted by placing the dot half-way between the integers on either side.

Display the numbers 1 , -2 , $4\frac{1}{2}$, -5.5 , 9 , 0 on a number line.



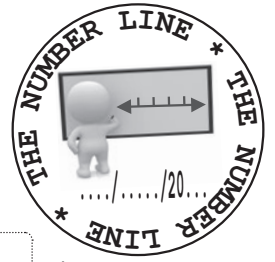
Write these numbers in **ascending** order.

Read the numbers from left to right (\rightarrow)

\therefore The numbers in ascending order are: $-5.5, -2, 0, 1, 4\frac{1}{2}, 9$



The number line



1 Insert the correct symbol $<$ (less than) or $>$ (greater than) for each of these.

a $6 \square 4$

b $3 \square -12$

c $-5 \square 4$

d $-11 \square -2$

e $8 \square -8$

f $-7\frac{1}{2} \square -7$

g $-\frac{1}{2} \square 0$

h $9\frac{1}{2} \square 9.9$

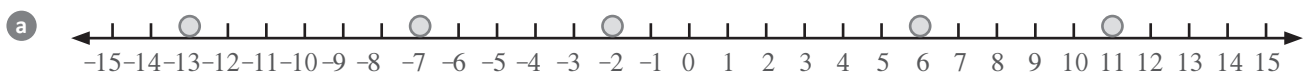
i $-5.1 \square -5.8$

j $-1.8 \square 1.2$

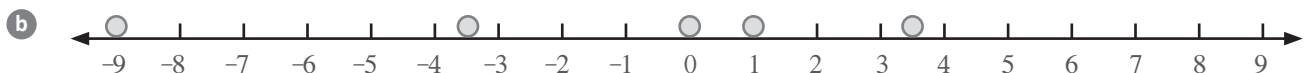
k $-12.5 \square -21.5$

l $11\frac{1}{2} \square 11\frac{1}{3}$

2 List the numbers displayed below in ascending order

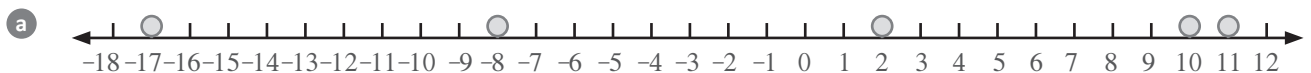


.....,,,,



.....,,,,

3 List the numbers displayed below in descending order



.....,,,,



.....,,,,



The number line



$$\frac{1}{2} = 0.5$$

4 Display the following sets of numbers on a number line.

- a $-5, 8, -1, 3, 6$.



- b $-8.5, 1.5, -2, 7.5, 5$.



- c $-4\frac{1}{2}, 4, 6\frac{1}{2}, -\frac{1}{2}, 5$.

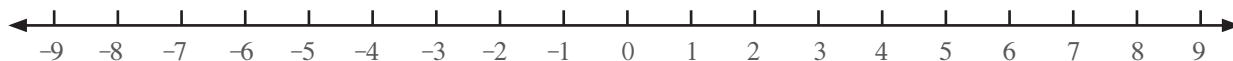


- d $-7, 7.5, 3\frac{1}{2}, 0, 2\frac{1}{2}, 2$.

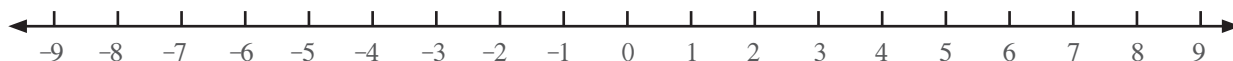


5 Display the following sets of numbers on a number line.

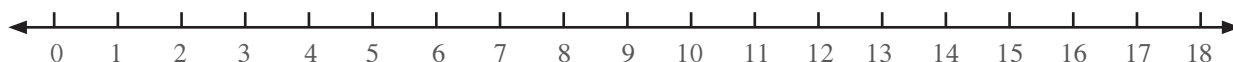
- a The first six **odd** integers above -8 .



- b All the **negative even** integers less than 7 but greater than negative 8.



- c All the **multiples of 3** higher than 7 but lower than 18.

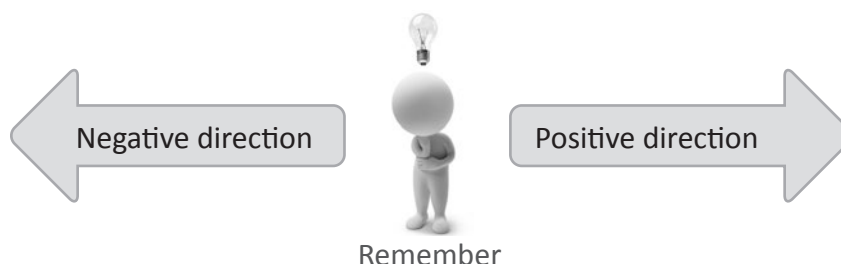


- d All the **integers** that are 3 or 7 whole numbers away from -2 .



Addition and subtraction using a number line

Starting from zero each time, calculations involving directed numbers can be made using a number line.

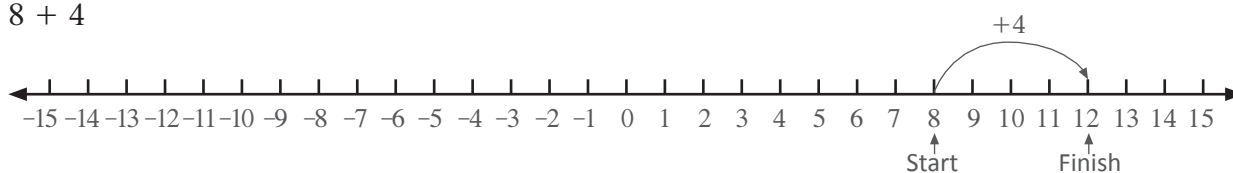


Remember

Always start from 0 and move using each directed number one at a time.

Use a number line to calculate these:

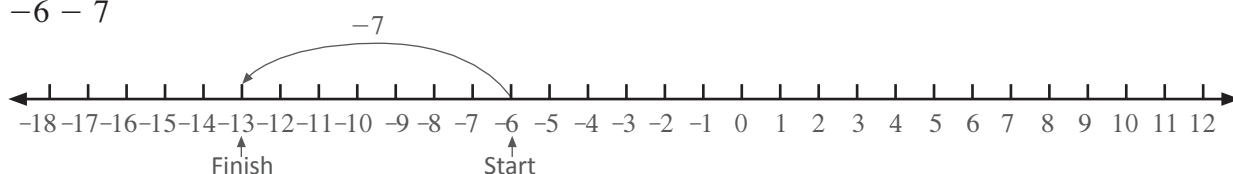
(i) $8 + 4$



Starting at +8, 4 to the right finishes at 12 right (+12)

$\therefore 8 + 4 = 12$

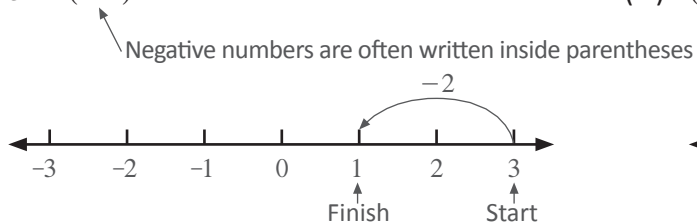
(ii) $-6 - 7$



Starting at -6, then 7 to the left finishes at 13 left (-13)

$\therefore -6 - 7 = -13$

(iii) $3 + (-2)$

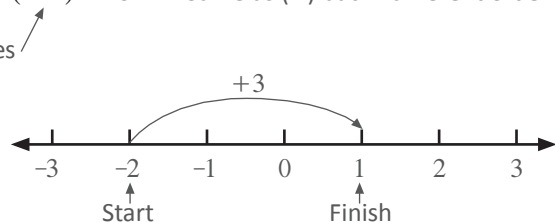


Starting at +3 then 2 to the left finishes at 1 right (+1)

$\therefore 3 + (-2) = 1$

(iv) $(-2) + 3$

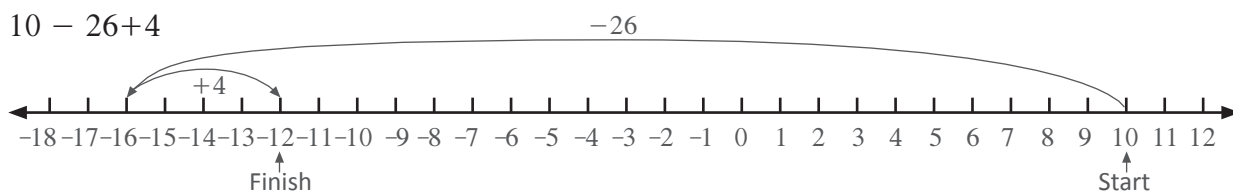
Same as (iii) but in different order



Starting at -2, 3 to the right finishes at 1 right (+1)

$\therefore (-2) + 3 = 1$

(v) $10 - 26 + 4$



Starting at +10, then 26 to the left followed by 4 to the right finishes at 12 left (-12)

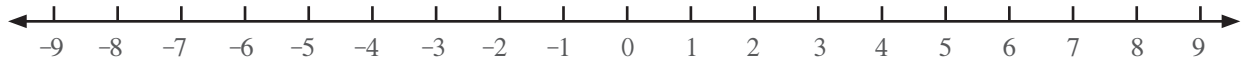
$\therefore 10 - 26 + 4 = -12$



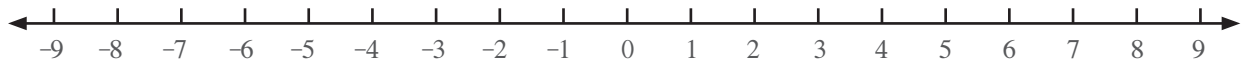
Addition and subtraction using a number line

1 Show the calculation for each of these on a number line and write down the answer.

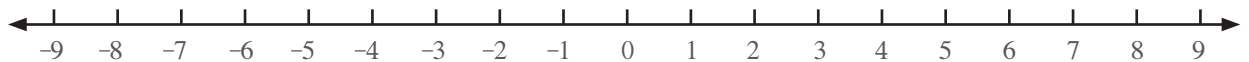
a $3 + 5 =$



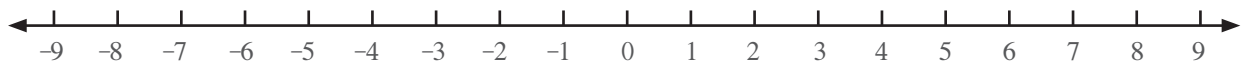
b $(-4) + 8 =$



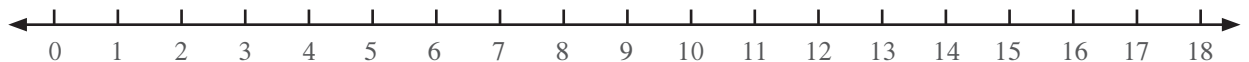
c $(-3) - 1 =$



d $7 - 15 =$

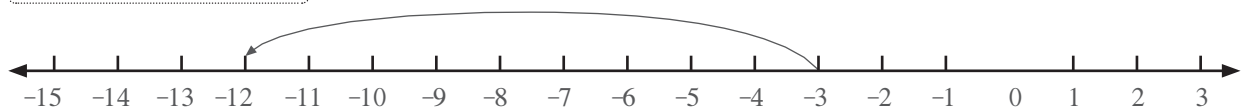


e $14 - 6 + 3 =$

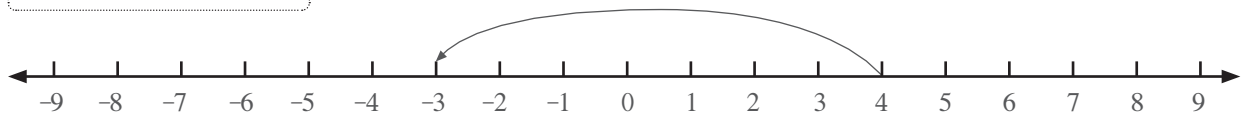


2 Write down the calculation to get the given answer shown on each of these number lines:

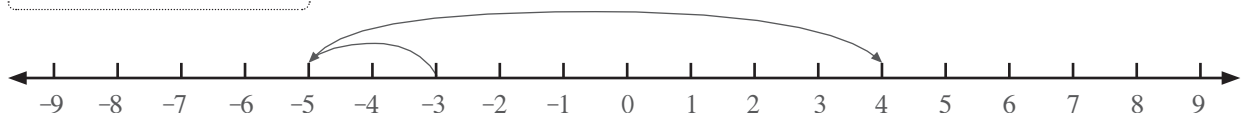
a $= -12$



b $= -3$



c $= 4$

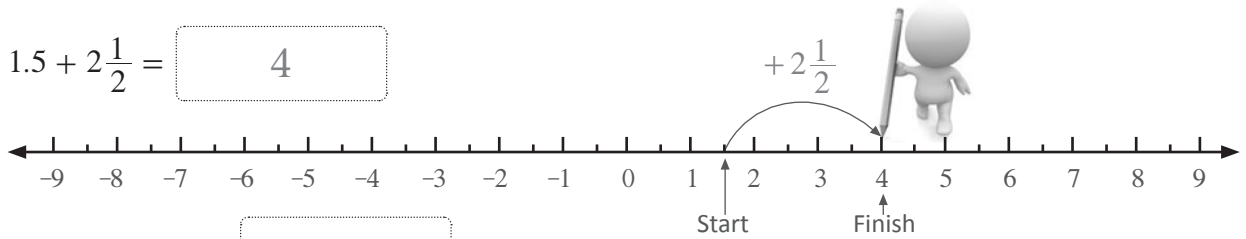




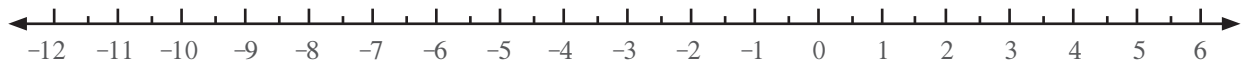
Addition and subtraction using a number line

3 Show the calculation for each of these on a number line and state the answer.

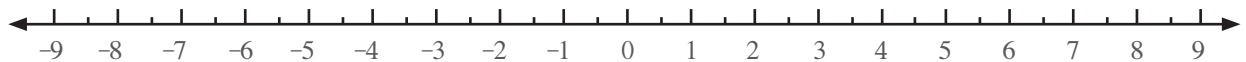
a $1.5 + 2\frac{1}{2} =$



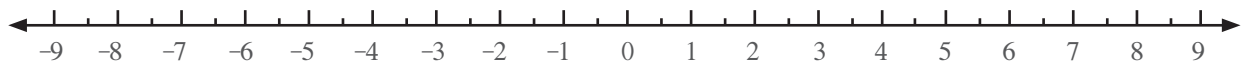
b $(-2.5) - 4.5 =$



c $-4 + 10.5 =$



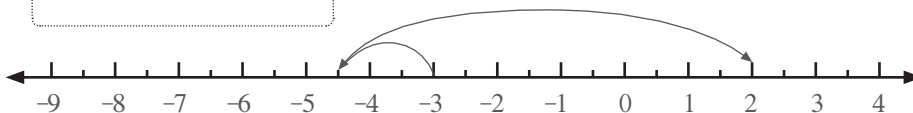
d $8\frac{1}{2} - 5.5 =$



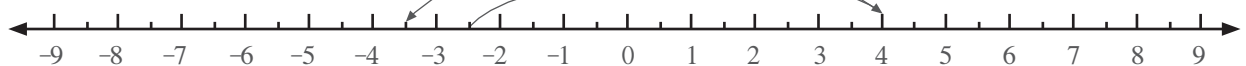
Try these trickier ones!

4 Write down the calculation shown on each of these number lines:

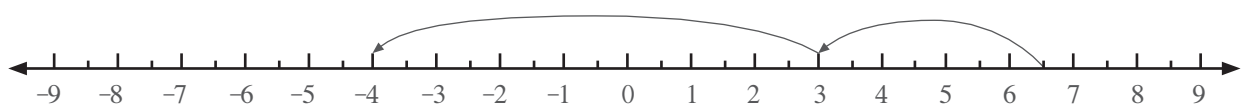
a = 2



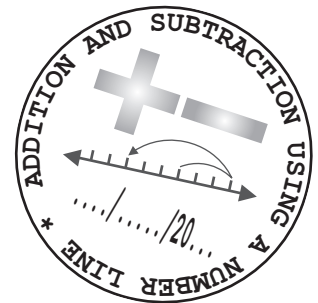
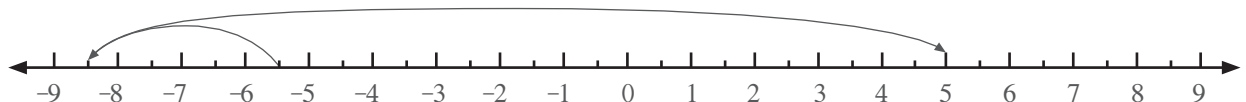
b = -3.5



c =



d =





The mysterious Dr Thermos!

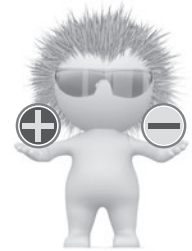
Dr Thermos controls the temperature of his laboratory using large hot or cold spheres.



+1°C
Hot Sphere





-1°C
Cold Sphere



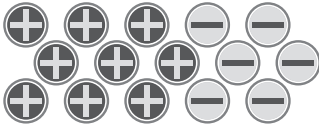


Count the number of each sphere type to find the overall temperature of his laboratory.

Calculate the temperature of the laboratory for these sphere combinations:

- (i)  7 hot spheres and 3 cold spheres
 $\therefore +7^{\circ}\text{C}$ and -3°C
 \therefore laboratory is $+4^{\circ}\text{C}$ $+7$ and -3 combine to make $+4$
- (ii)  2 hot spheres and 3 cold spheres
 $\therefore +2^{\circ}\text{C}$ and -3°C
 \therefore laboratory is -1°C $+2$ and -3 combine to make -1

To change the temperature, Dr Thermos adds or removes spheres.

Show two ways Dr Thermos can raise the temperature by $+2^{\circ}\text{C}$ for the combination of spheres below.

-  9 hot spheres and 6 cold spheres
 $\therefore +9^{\circ}\text{C}$ and -6°C
 \therefore laboratory is currently $+3^{\circ}\text{C}$ $+9$ and -6 combine to make $+3$
- (i) Dr Thermos could **add** two **hot** spheres
 9 hot spheres + 2 hot spheres and 6 cold spheres
 $\therefore +11^{\circ}\text{C}$ and -6°C
 \therefore laboratory is now $+5^{\circ}\text{C}$
 \therefore a change of $+2^{\circ}\text{C}$ $+11$ and -6 combine to make $+5$
- (ii) Dr Thermos could **remove** two **cold** spheres
 9 hot spheres and 6 cold spheres - 2 cold spheres
 $\therefore +9^{\circ}\text{C}$ and -4°C
 \therefore laboratory is now $+5^{\circ}\text{C}$
 \therefore a change of $+2^{\circ}\text{C}$ $+9$ and -4 combine to make $+5$



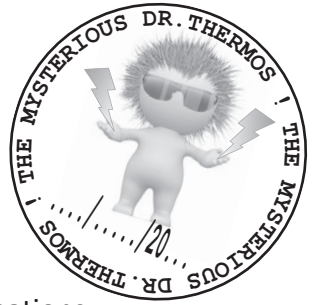
The mysterious Dr Thermos!



$+1^{\circ}\text{C}$
Hot Sphere



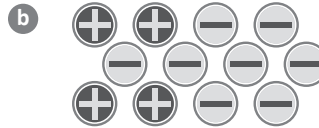
-1°C
Cold Sphere



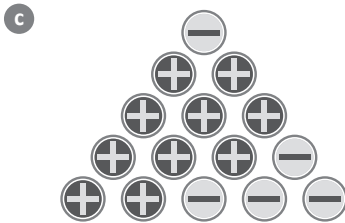
- 1 Write down the temperature of Dr Thermos' laboratory for these sphere combinations:



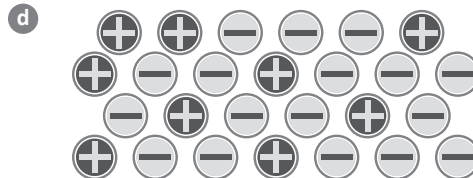
Laboratory temperature: $^{\circ}\text{C}$



Laboratory temperature: $^{\circ}\text{C}$



Laboratory temperature: $^{\circ}\text{C}$



Laboratory temperature: $^{\circ}\text{C}$

- 2 Write the number and type of hot or cold spheres that would need to be added to the laboratory to cause these changes in temperature.

a $+3^{\circ}\text{C}$

b -9°C

3 hot spheres



c -6°C

d $+20^{\circ}\text{C}$

- 3 Write the number and type of hot or cold spheres that would need to be removed from the laboratory to cause these changes in temperature.

a -7°C

b $+2^{\circ}\text{C}$

c -5°C

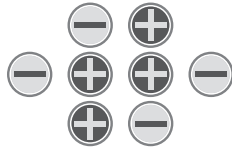
d $+8^{\circ}\text{C}$



The mysterious Dr Thermos!

- 4 For each of these combinations of hot and cold spheres, explain two different ways Dr Thermos can change the temperature of the laboratory to equal the amount given in the square brackets.

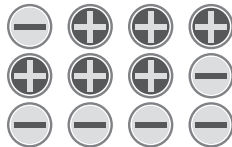
a $[+4^{\circ}\text{C}]$



(i) First way:

(ii) Second way:

b $[-2^{\circ}\text{C}]$



(i) First way:

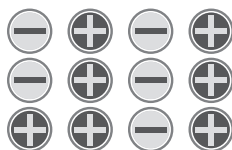
(ii) Second way:

Try these trickier ones!

- 5 If Dr Thermos could only use the spare spheres shown below each given combination, explain two different ways he could change the temperature of the laboratory to equal the amount in the square brackets.

You must use all of the spare spheres for at least one of the methods

a $[+6^{\circ}\text{C}]$



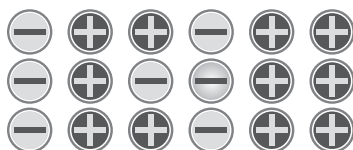
Spare spheres: +, +

(i) First way:

(ii) Second way:

Using all the spare spheres

b $[-1^{\circ}\text{C}]$



Spare spheres: -, -

(i) First way:

(ii) Second way:

Using all the spare spheres



The mysterious Dr Thermos!

Puzzle time!

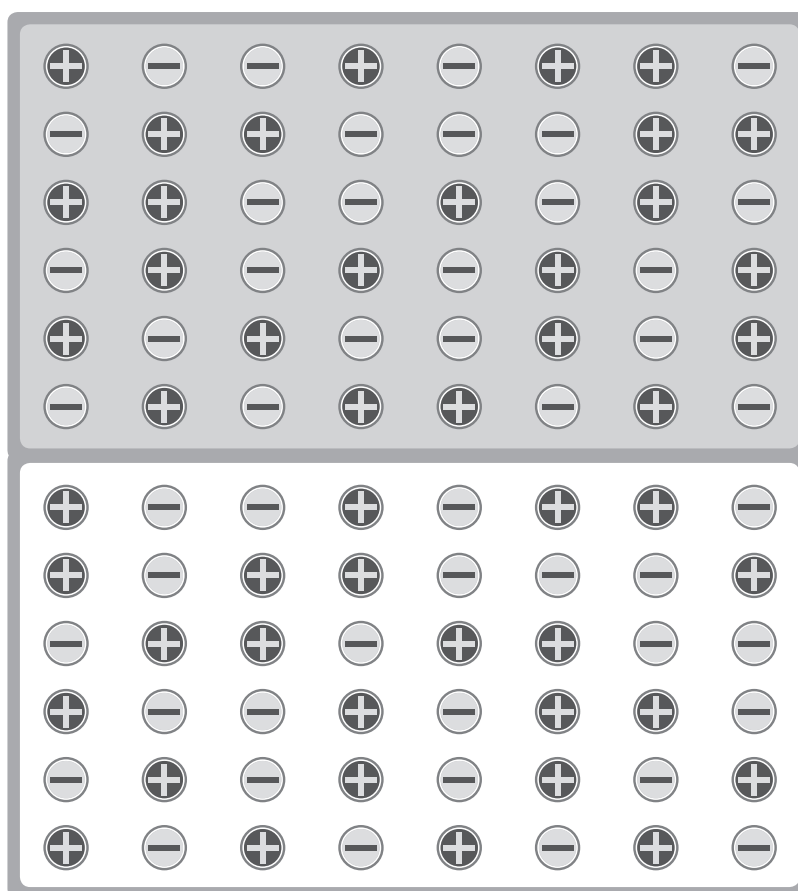
- 6 Dr Thermos has a puzzle room at the entrance to his laboratory to keep his experiments secret. The door will open when the display in the puzzle room reads -2°C . The display currently reads $+3^{\circ}\text{C}$.

- Each hot ($+1^{\circ}\text{C}$) or cold (-1°C) step can be used only once.
- Only sideways (\longleftrightarrow) or forward (\uparrow) steps can be taken.
- Steps in the white section are switched on (**added** to the puzzle room).
- Steps in the grey section are switched off (**removed** from the puzzle room).

- Draw a pathway that will allow you to enter the laboratory of Dr Thermos.
- See if you can find another path!

$+3^{\circ}\text{C}$

Finish anywhere along here



Start anywhere along here

Adding and subtracting directed numbers

The exercises in Dr Thermos' laboratory were examples of adding and subtracting directed numbers.

Here is what happened:

$$\oplus = +1^{\circ}\text{C}$$

$$\ominus = -1^{\circ}\text{C}$$

Addition	{	When a \oplus was added, the temperature went up
		When a \ominus was added, the temperature went down
subtraction	{	When a \oplus was removed (Subtracted), the temperature went down
		When a \ominus was removed (Subtracted), the temperature went up

The rules for adding and subtracting represent the same thing.

Use the pattern above to calculate these:

$$\oplus = +1 \text{ and } \ominus = -1$$

Addition	{	$5 + \oplus = 5 + (+1) = 5 + 1 = 6$	$+$	$+$	$=$	$+$	(Same signs together means plus)
		$5 + \ominus = 5 + (-1) = 5 - 1 = 4$	$+$	$-$	$=$	$-$	(Opposite signs together means minus)
Subtraction	{	$5 - \oplus = 5 - (+1) = 5 - 1 = 4$	$-$	$+$	$=$	$-$	(Opposite signs together means minus)
		$5 - \ominus = 5 - (-1) = 5 + 1 = 6$	$-$	$-$	$=$	$+$	(Same signs together means plus)

These rules work for all kinds of directed additions and subtractions

Calculate these additions and subtractions:

(i) $3 + (-2) = 3 - 2 = 1$ $+$ $-$ $=$ $-$ (ii) $(-2) + 3 = -2 + 3$ Same as (i) but in a different order
 $= 1$ $= 1$

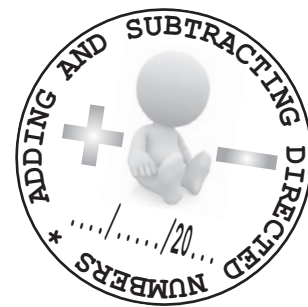
(iii) $6 - (-2.5) = 6 + 2.5 = 8.5$ $-$ $-$ $=$ $+$ (iv) $(-11) - (-4) = -11 + 4$ $-$ $-$ $=$ $+$
 $= 8.5$ $= -7$

(v) $2\frac{1}{2} + (-3) = 2\frac{1}{2} - 3 = -\frac{1}{2}$ $+$ $-$ $=$ $-$ (vi) $(-12.2) + (-5.2) = -12.2 - 5.2$ $+$ $-$ $=$ $-$
 $= -\frac{1}{2}$ $= -17.4$

(vii) $9.5 - (-8.5) + 6 = 9.5 + 8.5 + 6 = 24$ (viii) $4\frac{1}{3} + (-1) - (-6\frac{2}{3}) = 4\frac{1}{3} - 1 + 6\frac{2}{3} = 10$

**Adding and subtracting directed numbers**

Calculate these additions and subtractions of integers without a calculator.



1 **a** $-5 + 3$

b $-4 + 7$

c $-4 - 4$

d $-11 - 8$

e $1 - 16$

f $16 - 22$

g $6 + (-3)$

h $30 + (-17)$

i $0 + (-8)$

j $-4 + (-5)$

k $-10 + (-19)$

l $13 - (-5)$

m $20 - (-21)$

n $-18 - (-6)$

o $-15 - (-26)$

2 **a** $4 - 8 + 2$

b $-9 + 6 - 8$

c $-3 + 4 - 8 - 7$

d $-7 + (-4) + 13$

e $2 - (-6) - 7$

f $5 + (-1) - (-14)$



Adding and subtracting directed numbers

Simplify these additions and subtractions:

Use the number line on the side if it helps.

3 a $5.5 - 8$

b $-3 + 2.5$

c $-2 - 7.5$

d $-6\frac{1}{2} - 19$

e $9.5 - 16$

f $7.5 + (-18)$

g $0.5 + (-5.5)$

h $-12\frac{1}{2} - (-3\frac{1}{2})$

i $0 - (+8) - (-6)$

4 a $-2 - 7 + 3.5$

b $1 + 0.5 - 6$

c $2 + (-3.5) + 1.5$

d $-7\frac{1}{2} + (-9) - (-2\frac{1}{2})$

e $-8.5 - (-6\frac{1}{2}) + 18$

f $-4 + (-4) - (-4\frac{1}{4}) - 4\frac{1}{4}$





Adding and subtracting directed numbers

For trickier questions, the calculator can be used.

Use the **(-)** button to change between positive and negative when entering numbers into the calculator.

Sometimes the **(-)** button may look like **+/-**

$$-29.3 - (-42.9) + (-19.6) = -29.3 + 42.9 - 19.6$$

Simplify using the rules

(-) **2** **9** **.** **3** **+** **4** **2** **.** **9** **-** **1** **9** **.** **6** **=**
 $= -6$

Enter into calculator



Use your calculator (only if you need to) for these:

5 a $53 - 89$

b $-43 + 94$

c $34 + (-51)$

d $-25 + (-94)$

e $-94 - (-28)$

f $-16 + (-87) - (-41)$

g $-2.6 + 10.4$

h $-0.5 - 6.7$

i $-10.3 + (-5.6)$

j $66 - 34.6 + (-24.4)$

k $1.06 + 4.5 - 9.7$

l $-0.23 + 1.76$

m $(-71.23) + (-52.38)$

n $-1.204 - (-5.093)$

o $98.23 - (-13.8) - 112.3$

p $-29\frac{1}{3} + 51$

q $-30\frac{1}{3} - (-66\frac{1}{2})$

r $100 + (-54\frac{1}{2}) - 46\frac{1}{4}$

Multiplying and dividing directed numbers

The rules for multiplying and dividing are similar to adding and subtracting:

Multiplication	Positive	×	Positive	=	Positive	$+$	×	$+$	=	$+$	(Same signs mean answer is positive)
	Positive	×	Negative	=	Negative	$+$	×	$-$	=	$-$	(Opposite signs mean answer is negative)
	Negative	×	Positive	=	Negative	$-$	×	$+$	=	$-$	(Opposite signs mean answer is negative)
	Negative	×	Negative	=	Positive	$-$	×	$-$	=	$+$	(Same signs mean answer is positive)

Division { The exact same rules apply because division is the opposite operation to multiplication!

These rules always apply to all multiplication and division calculations.

Calculate these multiplications and divisions:

$$(i) \quad -4 \times 3 = -12 \quad - \times + = - \quad (ii) \quad -14 \div 2 = -7 \quad - \div + = -$$

$$(iii) \quad 12 \times (-1.5) = -18 \quad + \times - = - \quad (iv) \quad 24 \div (-4.8) = -5 \quad + \div - = -$$

$$(v) \quad -8 \times (-3) = 24 \quad - \times - = + \quad (vi) \quad -46.8 \div (-18) = 2.6 \quad - \div - = +$$

$$(vii) \quad -3\frac{1}{2} \times 6 = -21 \quad - \times + = - \quad (viii) \quad -32 \div \left(-3\frac{1}{5}\right) = 10 \quad - \div - = +$$

Be careful squaring (or cubing) directed numbers as parentheses affect the question.

$$(ix) \quad (-3)^2 = (-3) \times (-3) = 9 \quad (x) \quad -3^2 = -(3 \times 3) = -9$$

If more than two terms are multiplied or divided, simplify by calculating in order from left to right.

Simplify these mixed questions:

$$(i) \quad 6 \times (-3) \div 9 \quad = -18 \div 9 \quad \text{Calculate } 6 \times (-3) \text{ first} \quad + \times - = - \\ = -2 \quad - \div + = -$$

$$(ii) \quad (-2) \div 0.5 \times (-3) \quad = -4 \times (-3) \quad \text{Calculate } (-2) \div 0.5 \text{ first} \quad - \div + = - \\ = 12 \quad - \times - = +$$

$$(iii) \quad (-1)^3 \quad = (-1) \times (-1) \times (-1) \quad \text{Calculate } (-1) \times (-1) \text{ first} \quad - \div - = + \\ = 1 \times (-1) \quad \text{Calculate } 1 \times (-1) \text{ next} \quad + \div - = - \\ = -1$$

$$(iv) \quad 8\frac{4}{5} \times 5 \div (-2) \div (-11) \quad = 44 \div (-2) \div (-11) \quad \text{Calculate } 8\frac{4}{5} \times 5 \text{ first} \quad + \times + = + \\ = -22 \div (-11) \quad \text{Calculate } 44 \div (-2) \text{ next} \quad + \div - = - \\ = 2 \quad - \div - = +$$

**Multiplying and dividing directed numbers**

1 Simplify these without using a calculator:

a $3 \times (-7)$

b -6×8

c $-36 \div 12$

d $48 \div (-4)$

e $-16 \times (-1)$

f $-2 \times (-2)$

g $-12 \div (-12)$

h $21 \div (-21)$

i $3 \times (-4) \div 6$

2 Simplify these without using a calculator:

a $-24 \div 3 \div (-8)$

b $-18 \div (-6) \times 4$

c $-5 \times 2 \times (-1) \times 2$

d -5^2

e $(-4)^2$

f $(-3)^3$

g -2^3

h $(-2)^2 \div 4$

i $(-3)^2 \times -3$

j $6^3 \div (-2)$

k $2^4 \div -4^2$

l $9^2 \div -3^2 \times (3)$



Multiplying and dividing directed numbers



3 Simplify these using a calculator:

a $27 \times (-4)$

b -21×13

c -4×29

d $-95 \div (-19)$

e $125 \times (-49)$

f $-162 \div 18$

g -53×16

h $(-391) \div (-23)$

i $-15 \times 15 \div (-3)$

4 Use your calculator to simplify these:

a $-25 \times 13 \div (-65)$

b $-500 \div (-5) \div 25$

c $-552 \div (-23) \times (-8)$

d $2\frac{1}{2} \times (-6) \div 5$

e $12.5 \times 25 \div (-0.25)$

f $-1\frac{2}{5} \div (-14) \times 5 \times (-2)$

g $-4 \times 3.6 \times 22 \div 7.92$

h $-3404.7 \div (-3.6) \div 1\frac{1}{2} \div 9.7$

i $260 \div (-26) \times 2\frac{3}{5} \div 0.26$

j $(-7)^3$

$(-7) x^3 =$

-343

k -2^{10}

$(-2) x^{10} =$

-1024

l $(-8)^3$

m $(-9)^4$

n $-(-6)^3$

o $-(-1.5)^5$





Combining the basic operations

Questions that mix multiplying/dividing with adding/subtracting need to be done in a certain order.

Multiplication or division operations are ranked the highest and therefore must be completed first.

Calculate these combined operations questions:

(i) $-2 + 3 \times 5$

$$\begin{array}{l} -2 + \underbrace{3 \times 5}_{\text{Do this first}} = -2 + 15 \\ = 13 \end{array}$$

Add -2 and 15

(ii) $35 \div (-5) - 1$

$$\begin{array}{l} \underbrace{35 \div (-5)}_{\text{Do this first}} - 1 = -7 - 1 \\ \phantom{\underbrace{35 \div (-5)} - 1} = -8 \end{array}$$

Subtract 1 from -7

If more than one multiply/divide sign, work left to right. Do the same for addition/subtraction.

Calculate these combined operations questions:

(i) $-28 \div 7 \times 3 + 6$

$$\begin{array}{l} \underbrace{-28 \div 7}_{\text{Do this first}} \times 3 + 6 = -4 \times 3 + 6 \\ \phantom{\underbrace{-28 \div 7} \times 3 + 6} = -12 + 6 \\ \phantom{\underbrace{-28 \div 7} \times 3 + 6} = -6 \end{array}$$

Multiply -12 by 3

Add -12 and 6

(ii) $17 - 4 - (-10) \times 2$

$$\begin{array}{l} 17 - 4 - \underbrace{(-10) \times 2}_{\text{Do this first}} = 17 - 4 - (-20) \\ = 13 - (-20) \\ = 33 \end{array}$$

Subtract 4 from 17

Subtract -20 from 13 ($= 13 + 20$)

(iii) $24 \div 3 + (-5) \times 4$

$$\begin{array}{l} \underbrace{24 \div 3}_{\text{Do this first}} + (-5) \times 4 = 8 + (-5) \times 4 \\ \phantom{\underbrace{24 \div 3} + } = 8 + (-20) \\ \phantom{\underbrace{24 \div 3} + } = -12 \end{array}$$

Multiply -5 by 4

Add 8 and -20

(iv) $6 \times (-3) - 210 \div 30 \times (-2)$

$$\begin{array}{l} \underbrace{6 \times (-3)}_{\text{Do this first}} - 210 \div 30 \times (-2) = -18 - 210 \div 30 \times (-2) \\ \phantom{\underbrace{6 \times (-3)} - } = -18 - 7 \times (-2) \\ \phantom{\underbrace{6 \times (-3)} - } = -18 + 14 \\ \phantom{\underbrace{6 \times (-3)} - } = -4 \end{array}$$

Divide -210 by 30

Multiply -7 by -2

Add -18 and 14

**Combining the basic operations**

1 Simplify these combined operations without a calculator:

a $-4 + 16 \div 8$

b $6 \times (-8) - 12$

c $50 \div (-25) + 10$

d $-8 - 5 \times 4$

e $5 + 7 \times (-6)$

f $-11 \times -2 + 18$

g $22 - 39 \div (-3)$

h $-10 - 4 \times 15$

2 Simplify these combined operations without a calculator:

a $4 \times (-2) + 18 - 11$

b $-3 \times 6 \div 9 + 3$

c $-9 + 48 \div (-12) + 13$

d $32 \div 4 - 3 \times (-4)$

3 Use a calculator to simplify these combined operations:

a $28 \div (-4) + 13 \times 7$

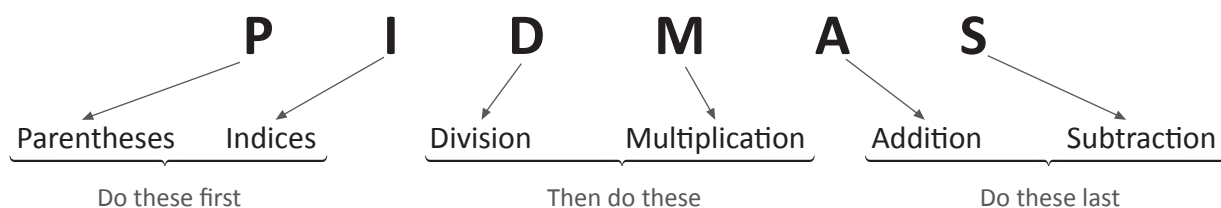
b $120 \div (-15) \times 24 - 8$

c $-25 \times 6 - 224 \div (-32)$

d $30 \times (-15) - 35 \div 4 \times (-16)$

Order of operations

For more complex calculations involving parentheses and indices, we use a similar convention:



Simplify these directed number calculations using the correct order of operations:

(i) $(12 - 8) \times 7$

$$(12 - 8) \times 7 = 4 \times 7$$

$$= 28$$

Subtract 8 from 12 inside the parentheses
Multiply 4 and 7

(ii) $-21 \div (2 + 5) + 3$

$$-21 \div (2 + 5) + 3 = -21 \div 7 + 3$$

$$= -3 + 3$$

$$= 0$$

Add 2 and 5 inside the parentheses
Divide -21 by 7
Add -3 and 3

(iii) $(-4)^2 \div (8 - 6)$

$$(-4)^2 \div (8 - 6) = (-4)^2 \div 2$$

$$= 16 \div 2$$

$$= 8$$

Subtract 6 from 8 inside the parentheses
Square -4
Divide 16 by 2

(iv) $15 \times (-5 + 3)^3 - 20$

$$15 \times (-5 + 3)^3 - 20 = 15 \times (-2)^3 - 20$$

$$= 15 \times (-8) - 20$$

$$= -120 - 20$$

$$= -140$$

Add -3 and 5 inside the parentheses
Cube -2
Multiply 15 by -8
Subtract 20 from -120

(v) $29 - (13 - 4 \times 5) + 3^2$

$$29 - (13 - 4 \times 5) + 3^2 = 29 - (13 - 20) + 3^2$$

$$= 29 - (-7) + 3^2$$

$$= 29 - (-7) + 9$$

$$= 36 + 9$$

$$= 45$$

Multiply -4 by 5 inside the parentheses
Subtract 20 from 13 inside the parentheses
Square 3
Subtract -7 from 29
Add 36 and 9

**Order of operations**

1 Simplify these directed number calculations using the correct order of operations without a calculator:

a $(16 - 10) \times 4$

b $34 \div (6 + (-8))$

c $-5 \times (27 \div 9)$

d $(-13 - (-19)) \times 3$

e $-5 \times (14 - 9) - 5$

f $4 \times (36 \div 2) - (-4)$

g $18 + (16 \div 4 + 10)$

h $(18 \div 2 - (-4)) - 7$

2 Simplify these calculations containing indices using the correct order of operations without a calculator:

a $(-2)^2 \div (-2)$

b $(-7 - 5) \times (-1)^3$

c $(8 - 14)^2 - 16$

d $(3^3 - 43) \div 8$

**Order of operations**

3 Try simplify these trickier ones using the correct order of operations:

a $26 \div (14 + 2 \times 6) + (-1)^2$

b $-100 \times (12 - 32 \div (-4)) \div 40$

c $(11 \times 2^3 - 10) \div 39 + 8$

d $9 \times ((-8) - 8 \div 4 + 16) - 4^3$

e $200 \div (-4) \times (-3) \div (16 - 14 \times (-1)^3)$



Order of operations with grouping symbols

For questions containing multiple pairs of grouping symbols, complete each pair separately.

Grouping symbol names: () = parentheses [] = square brackets { } = braces

Simplify these combined operations questions containing multiple brackets:

(i) $(10 \times 2) \div (-7 - 3)$

$$(10 \times 2) \div (-7 - 3) = 20 \div (-10) \\ = -2$$

Simplify the parentheses

Divide 20 by -10



The same question can be written as a fraction.

$$\frac{10 \times 2}{-7 - 3} = (10 \times 2) \div (-7 - 3)$$

$$\frac{\text{numerator}}{\text{denominator}} = (\text{numerator}) \div (\text{denominator})$$

(ii) $\frac{34 - 3 \times 4}{22 \div (-2)}$

$$\frac{34 - 3 \times 4}{22 \div (-2)} = (34 - 3 \times 4) \div (22 \div (-2)) \\ = 22 \div (-11) \\ = -2$$

Re-write with parentheses

Simplify the parentheses

Divide 22 by -11

When questions contain brackets within other brackets, work from the inside to the outside.

Simplify these combined operations questions containing multiple brackets:

(i) $3 \times [(1 - 6) \div 5]$

$$3 \times [(1 - 6) \div 5] = 3 \times [-5 \div 5] \\ = 3 \times -1 \\ = -3$$

Simplify the parentheses first

Simplify the square brackets

Multiply 3 by -1

(ii) $30 - \{18 - [26 \div (2^2 - 17)]\}$

$$30 - \{18 - [26 \div (2^2 - 17)]\} = 30 - \{18 + [26 \div (4 - 17)]\} \\ = 30 - \{18 + [26 \div (-13)]\} \\ = 30 - \{18 + -2\} \\ = 30 - 16 \\ = 14$$

Simplify the parentheses

Simplify the square brackets

Simplify the braces

Subtract 16 from 30



Order of operations with grouping symbols



1 Simplify these questions:

a $(16 + 9) + (14 - 7)$

b $(1 - 9) - (3 - 8)$

c $(7 - 11) \times (4 + 2)$

d $(18 - 3) \div (-12 - (-7))$

e $\frac{46 - 19}{5 - 2}$

$= (46 - 19) \div (5 - 2)$

$= 27 \div 3$

$= 9$



f $\frac{-2 \times 10}{48 \div 12}$

g $(50 + 54 \div 9) \div (-24 \div 3)$

h $\frac{8 \times 5 - 12}{42 \div (-6)}$

2 Simplify these questions with multiple grouping symbols:

a $-12 \div [(7 + 11) \div 9]$

b $[10 + (-14 \div 2)] \times 8$

c $[-6 \times (13 - (-7))] \div 4$

d $5 \times [(2^2 - 9) \times 3]$



Order of operations with grouping symbols

3 Simplify these:

a $\{[12 + (6 \times 8)] \div 3\} - 25$

b $(-2)^3 + \{-2 + [2 \times (2 + 2)]\}$

4 Give these three tricky ones a go to earn an awesome stamp.

a $\frac{(14 - 8)^2 \times (-1)^3}{15 - (-3)}$

b $-4 \times \{-72 + (-6)^2 \div [(4 \times (-5)) - (-2)]\}$

c $-3 \times \{[(-3 - (-3))^2 + (-3)^2] \div 3 \times (-3)\}$





Here is a summary of the important things to remember for directed numbers

Types of numbers

- **Positive and negative** numbers are numbers that have both size and direction.
Eg: $+2$ or -3
- **Rational** numbers can be written as a fraction. They include whole numbers and terminating or recurring decimals. Eg: 8 , 2.3 , $4.\dot{5}$, $\frac{1}{2}$, $\sqrt{4}$.
- **Irrational** numbers cannot be written as a fraction. These include decimals that keep going without following a pattern, and roots of numbers that are not perfect squares/cubes etc.
Eg: $\sqrt{3}$, $1.2938164928\dots$, π .
- **Integers** are whole numbers only. They are rational and have no decimal or fraction parts.
Eg: 3 , -4 , 100 .

Magnitude and direction of numbers

- **Directed numbers** are positive (+) or negative (−) whole numbers.
- They show both the size (**magnitude**) and **direction** of values associated with change.
- It is important to first decide what will be the positive direction.

Ascending and descending order

Ascending order = lowest to highest

Descending order = highest to lowest

Adding and subtracting directed numbers

Addition	$+$	$+$	$=$	$+$	(Same signs together means plus)
	$+$	$-$	$=$	$-$	(Opposite signs together means minus)
Subtraction	$-$	$+$	$=$	$-$	(Opposite signs together means minus)
	$-$	$-$	$=$	$+$	(Same signs together means plus)

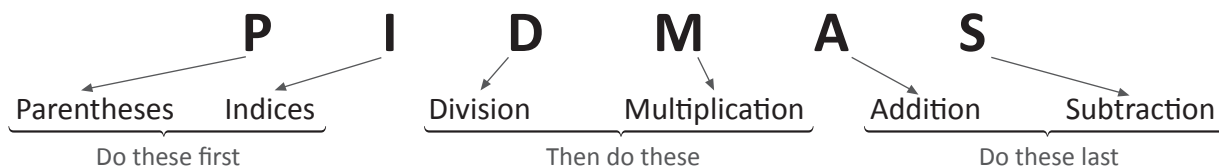
Multiplying and dividing direct numbers

Multiplication	Positive	\times	Positive	$=$	Positive	$+$	\times	$+$	$=$	$+$	(Same signs mean answer is positive)
	Positive	\times	Negative	$=$	Negative	$+$	\times	$-$	$=$	$-$	(Opposite signs mean answer is negative)
	Negative	\times	Positive	$=$	Negative	$-$	\times	$+$	$=$	$-$	(Opposite signs mean answer is negative)
	Negative	\times	Negative	$=$	Positive	$-$	\times	$-$	$=$	$+$	(Same signs mean answer is positive)

Division { The exact same rules apply because division is the opposite operation to multiplication!

Order of operations

The word **PIDMAS** helps to remember the order in which we need to perform calculations.





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